Review 26 Infinite Series, Conv/Div Tests

1. n^{th} term test: If seq $\not\to 0$ the series div.

series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 \cdots$ {sum}

seq $a_1, a_2, a_3 \cdots$ {list}

2. Geometric: $a + ar + ar^2 + \cdots = \frac{a}{1-r}$ if $|r|<1$

if $|r| \geq 1$, series div

Apr 5-9:40 AM

3. p-series $\sum \frac{1}{n^p}$: $p>1$, conv; $p \leq 1$, div

4. ratio test: use with ! and/or exp.

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$: $L<1$ conv; $L>1$ div; $L=1$?

5. integral test

$\int_1^{\infty} a_x\,dx$ and $\sum_{n=1}^{\infty} a_n$ behave the same

Apr 5-9:46 AM

neg terms?

6. AST: IF 1. signs alternate 2. |terms| gets smaller 3. seq $\to 0$, then series conv.

remainder $\leq |a_{n+1}|$

$a_{n+1} > 0$, underestimate

$a_{n+1} < 0$, overestimate

7. Absolute Conv. If $\sum |a_n|$ conv then $\sum a_n$ conv

Conditional convergence: If $\sum |a_n|$ div but $\sum a_n$ conv

Apr 5-9:53 AM

8. Comparison Test

$0 \leq a_n \leq b_n$

If $\sum b_n$ conv then $\sum a_n$ conv

If $\sum a_n$ div then $\sum b_n$ div

9. Limit Comparison Test

If a_n and b_n grow at same rate ($\lim_{n\to\infty} \frac{a_n}{b_n} = L$, $0 < L < \infty$)

then $\sum a_n$ and $\sum b_n$ behave same

Apr 5-10:01 AM

I $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ p series $p=\frac{1}{2}$ series diverges

II $\sum_{n=0}^{\infty} \frac{2^n}{n!}$ $\lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} = \lim_{n\to\infty} \frac{2}{n+1} = 0$

conv.

III $\sum_{n=2}^{\infty} \frac{1}{\ln n} > \sum \frac{1}{n}$ div, harmonic

div by comparison

Apr 5-10:06 AM

conv by LCT

IV $\sum_{n=1}^{\infty} \frac{n^2+n}{3^n+5}$ $\sum \frac{n^2}{3^n}$ conv

$\lim_{n\to\infty} \frac{n^2+n}{3^n+5} \cdot \frac{3^n}{n^2} = 1$ $\lim_{n\to\infty} \frac{(n+1)^2}{3^{n+1}} \frac{3^n}{n^2} = \frac{1}{3}$

seq grow at same rate

Apr 5-10:14 AM

find the interval of convergence

$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n\,2^n}$ $\lim_{n\to\infty} \left| \frac{(x-1)^{n+1}}{(n+1)\,2^{n+1}} \cdot \frac{n \cdot 2^n}{(x-1)^n} \right|$

endpts

$x=3$ $\sum \frac{2^n}{n \cdot 2^n}$ harmonic, div

$\lim_{n\to\infty} \frac{|x-1|}{2} \frac{n}{n+1} = \frac{|x-1|}{2} < 1$

$|x-1| < 2$

$-2 < x-1 < 2$

$-1 \leq x < 3$

$x=-1$ $\sum \frac{(-2)^n}{n\,2^n}$

$\sum \frac{(-1)^n}{n}$ conv by AST

Apr 5-10:22 AM